

Effect of interaction with neutrons in matter on flavor conversion of super-light sterile neutrino with active neutrino

Wei Liao, Yuchen Luo and Xiao-Hong Wu

Institute of Modern Physics, School of Sciences

East China University of Science and Technology,

130 Meilong Road, Shanghai 200237, P.R. China

A super-light sterile neutrino was proposed to explain the absence of the expected upturn of the survival probability of low energy solar boron neutrinos. This is because this super-light sterile neutrino can oscillate efficiently with electron neutrino through a MSW resonance happened in Sun. One may naturally expect that a similar resonance should happen for neutrinos propagating in Earth matter. We study the flavor conversion of this super-light sterile neutrino with active neutrinos in Earth matter. We find that the scenario of the super-light sterile neutrino can easily pass through possible constraints from experiments which can test the Earth matter effect in oscillation of neutrinos. Interestingly, we find that this is because the naively expected resonant conversion disappears or is significantly suppressed due to the presence of a potential V_n which arises from neutral current interaction of neutrino with neutrons in matter. In contrast, the neutron number density in the Sun is negligible and the effect of V_n is effectively switched off. This enables the MSW resonance in Sun needed in oscillation of the super-light sterile neutrino with solar electron neutrinos. It's interesting to note that it is the different situation in the Sun and in the Earth that makes V_n effectively turned off and turned on respectively. This observation makes the scenario of the super-light sterile neutrino quite interesting.

PACS numbers: 14.60.Pq, 13.15.+g

Introduction

Among many candidates of sterile neutrino proposed in literature, a super-light sterile neutrino appears to be very interesting [1, 2]. With a mass squared difference with ν_1 at around $\Delta m_{01}^2 \approx (0.5 - 2) \times 10^{-5} \text{ eV}^2$ and a mixing with electron neutrino around $\sin^2 2\theta_{01} \approx 0.001 - 0.005$ [2, 3], this sterile neutrino can help to explain the absence of the upturn of the solar boron neutrino spectrum at energy $E_\nu \lesssim 4 \text{ MeV}$ [4–7] which is expected in the LMA MSW [8, 9] solution of the solar neutrino anomaly. This is achieved with the help of a MSW resonant conversion of this super-light sterile neutrino with solar electron neutrino when neutrino travels from the interior of the Sun to the outside [1, 2].

One may naturally expect that there might exist a resonance of flavor conversion between this sterile neutrino and electron neutrino when neutrinos propagate in Earth matter. If a resonance happen, the effective mixing angle between this sterile neutrino and electron neutrino can reach maximal and the oscillation phase would be around $V_e L \sin 2\theta_{01}$ which can be of order one for a long enough oscillation length L , e.g. for neutrinos crossing the core of the Earth. Hence, the probability of flavor conversion could be large in resonance region. In particular, this would lead to a suppression of the total flux of active neutrinos when neutrinos pass through the core of the Earth.

Since the mass squared difference Δm_{01}^2 is several to ten times smaller than the solar mass squared difference Δm_{21}^2 , the associated resonance may happen at an energy much lower than the energy of the 1 – 2 resonance which is around 100 – 200 MeV in Earth matter. In particular, this resonant conversion of sterile neutrino with electron neutrino may happen for high energy solar neutrinos, supernovae neutrinos and for low energy atmospheric neutrinos. Hence, constraints on this super-light sterile neutrinos may exist in low energy atmospheric neutrino data, supernovae neutrino data and the test of the Earth matter effect in solar neutrino data. In this article [10] we examine the oscillation of the super-light sterile neutrino with active neutrinos in the Earth matter. Interestingly, we find that the naively expected resonant conversion between this super-light sterile neutrino with active neutrino disappears or is significantly suppressed in the presence of a sizeable potential from the neutral current interaction with matter. This is different from the oscillation happened in the Sun for which the neutron number density is low and the effect from neutral current can be effectively neglected. Consequently, we find that

this super-light sterile neutrino passes the possible constraints from experiments testing neutrino oscillation in Earth matter.

In the following of the present article we will first give a brief review of the formalism and the convention of neutrino oscillation with a super-light sterile neutrino. Then we will discuss the reduction of the 4ν formalism of neutrino oscillation in Earth matter to a 3ν formalism at energy < 1 GeV. In this 3ν formalism we will discuss in detail, with the help of a baseline dependent average potential, the crossing of energy levels of neutrinos and the oscillation of the super-light sterile neutrino with active neutrinos. We will show how the naively expected resonant conversion of this super-light sterile neutrino and active neutrinos disappears in the presence of V_n , an effective potential coming from neutral current interaction with neutrons in matter. For the completeness of the discussion in this article, we will then discuss the description of the oscillation of super-light sterile neutrino using the baseline dependent average potential and show that the discussion for the level-crossing using this baseline dependent potential is indeed a valid and good description despite the fact that Earth matter has complicated density profile. Finally we conclude.

Super-light sterile neutrino and its oscillation in 4ν and 3ν formalism

In the presence of a super-light sterile neutrino, the Hamiltonian governing the oscillation of $\nu = (\nu_s, \nu_e, \nu_\mu, \nu_\tau)^T$, the neutrino in flavor base, is

$$H = U H_0 U^\dagger + V, \quad (1)$$

where

$$\begin{aligned} H_0 &= \frac{1}{2E} \text{diag}\{\Delta m_{01}^2, 0, \Delta m_{21}^2, \Delta m_{31}^2\}, \\ V &= \text{diag}\{0, V_e + V_n, V_n, V_n\}. \end{aligned} \quad (2)$$

$V_e = \sqrt{2}G_F n_e$ and $V_n = -\frac{1}{\sqrt{2}}G_F n_n$ where n_e and n_n are number densities of electron and neutron in matter. U is a 4×4 unitary matrix describing the mixing of neutrinos. Neglecting CP violating phases, it can be parameterized by

$$U = R(\theta_{23})R(\theta_{13})R(\theta_{12})R(\theta_{02})R(\theta_{01})R(\theta_{03}), \quad (3)$$

where $R(\theta_{ij})$ is a 4×4 rotation matrix with a mixing angle θ_{ij} appearing at i and j entries,

e.g.

$$R(\theta_{01}) = \begin{pmatrix} \cos \theta_{01} & \sin \theta_{01} & 0 & 0 \\ -\sin \theta_{01} & \cos \theta_{01} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (4)$$

and

$$R(\theta_{02}) = \begin{pmatrix} \cos \theta_{02} & 0 & \sin \theta_{02} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta_{02} & 0 & \cos \theta_{02} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (5)$$

$\theta_{12,13,23}$ are mixing angles governing the flavor conversion of solar neutrinos, reactor neutrinos at short baseline and atmospheric neutrinos separately and they have been measured in solar, atmospheric, long baseline and reactor neutrino experiments [11–14]. If $\theta_{0i} (i = 1, 2, 3)$ are all zero, the mixing matrix $U = R(\theta_{23})R(\theta_{13})R(\theta_{12})$ and it reproduces the PMNS mixing matrix without CP violating phase for three active neutrinos. For anti-neutrinos the Hamiltonian is (1) with V replaced by $-V$.

As observed in [2], the presence of a super-light sterile neutrino with $\Delta m_{01}^2 \approx (0.5 - 2) \times 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta_{01} \approx 0.001 - 0.005$ can lead to a further suppression of the flux of the solar electron neutrinos at energy around $\lesssim 4 \text{ MeV}$. Hence it provides an explanation of the absence of the upturn of the solar neutrino flux at this energy range. For a small but non-zero θ_{02} , similar phenomena would occur for solar electron neutrinos [2].

For a small but non-zero θ_{03} , solar electron neutrinos are basically not affected by it but a resonant $\nu_s - \nu_\tau$ (or $\bar{\nu}_s - \bar{\nu}_\tau$) oscillation of atmospheric neutrino is expected to happen for energy around $10 - 15 \text{ GeV}$ [2]. So the scenario with a non-zero θ_{03} can be tested in future atmospheric neutrino experiments and long baseline experiments.

In this article we study the effect of U_{s1} and U_{s2} (basically $\sin \theta_{01}$ and $\sin \theta_{02}$) in oscillation of neutrinos in Earth matter. For energy as high as $\gtrsim 1 \text{ GeV}$, oscillation of active neutrinos with sterile neutrino due to effects of U_{e1} and U_{e2} would be strongly suppressed since $\Delta m_{01}^2 \approx (0.5 - 2) \times 10^{-5} \text{ eV}^2$ and $\Delta m_{21}^2 \approx 7.5 \times 10^{-5} \text{ eV}^2$ and the oscillation lengths of $1 - 2$ and $0 - 1$ oscillation are all much longer than the diameter of the Earth for such

high energy. Instead, we concentrate on oscillation of neutrinos for energy < 1 GeV. In this energy range, the $2 - 3$ oscillation is pretty fast and consequently we can reduce the 4ν formalism to a 3ν formalism in the study of the resonant oscillation between sterile neutrino and active neutrino. We will see that this 3ν formalism is more convenient for later discussion.

The 3ν formalism can be achieved from (1) as follows. We can first rotate the Hamiltonian by $R(\theta_{23})$ and $R(\theta_{13})$. Introducing $\nu' = (\nu_s, \nu'_e, \nu'_\mu, \nu'_\tau)^T$, which is obtained by

$$\nu' = R(\theta_{23})R(\theta_{13})\nu, \quad (6)$$

we find that the Hamiltonian for ν' is

$$\begin{aligned} H' &= (R(\theta_{23})R(\theta_{13}))^\dagger H R(\theta_{23})R(\theta_{13}) \\ &= U' H_0 (U')^\dagger + V', \end{aligned} \quad (7)$$

where $U' = R(\theta_{12})R(\theta_{02})R(\theta_{01})R(\theta_{03})$ and

$$V' = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & V_e \cos^2 \theta_{13} + V_n & 0 & V_e \sin \theta_{13} \cos \theta_{13} \\ 0 & 0 & V_n & 0 \\ 0 & V_e \sin \theta_{13} \cos \theta_{13} & 0 & V_e \sin^2 \theta_{13} + V_n \end{pmatrix}. \quad (8)$$

Eq. (7) can be rewritten as

$$H' = \begin{pmatrix} \hat{H}' & S \\ S^\dagger & \frac{\Delta m_{31}^2}{2E} \cos^2 \theta_{03} + \frac{\Delta m_{01}^2}{2E} \sin^2 \theta_{03} + V_e \sin^2 \theta_{13} + V_n \end{pmatrix}. \quad (9)$$

\hat{H}' is a 3×3 matrix

$$\hat{H}' = \hat{U} \hat{H}'_0 \hat{U}^\dagger + \hat{V}'. \quad (10)$$

\hat{H}'_0 , \hat{V}' and \hat{U} are

$$\hat{H}'_0 = \text{diag}\left\{\frac{\Delta m_{01}^2}{2E} \cos^2 \theta_{03} + \frac{\Delta m_{31}^2}{2E} \sin^2 \theta_{03}, 0, \frac{\Delta m_{21}^2}{2E}\right\}, \quad (11)$$

$$\hat{V}' = \text{diag}\{0, V_e \cos^2 \theta_{13} + V_n, V_n\}, \quad (12)$$

$$\hat{U} = \hat{R}(\theta_{12})\hat{R}(\theta_{02})\hat{R}(\theta_{01}), \quad (13)$$

where $\hat{R}(\theta_{ij})$ is a 3×3 matrix with mixing angle appearing at i and j entries, e.g.

$$\hat{R}(\theta_{01}) = \begin{pmatrix} \cos \theta_{01} & \sin \theta_{01} & 0 \\ -\sin \theta_{01} & \cos \theta_{01} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (14)$$

and

$$\hat{R}(\theta_{12}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{12} & \sin \theta_{12} \\ 0 & -\sin \theta_{12} & \cos \theta_{12} \end{pmatrix}. \quad (15)$$

S is a 3×1 matrix

$$S = \hat{U} \begin{pmatrix} \frac{1}{2} \sin 2\theta_{03} \left(\frac{\Delta m_{31}^2 - \Delta m_{01}^2}{2E} \right) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{2} \sin 2\theta_{13} V_e \\ 0 \end{pmatrix}, \quad (16)$$

From (6) one can see that ν'_e is mainly ν_e and has a small component of ν_μ and ν_τ . It has a probability $\cos^2 \theta_{13}$ being ν_e and a probability $\sin^2 \theta_{13}$ being ν_μ and ν_τ . Disappearance of solar ν_e can be effectively studied by examining the oscillation of ν'_e to other neutrinos [2].

As is well known, 2 – 3 resonance happens for energy 5 – 7 GeV in the Earth and for energy < 1 GeV, $|\Delta m_{31}^2/(2E)| \gg V_e$. In the Earth, the neutron number density is roughly of the same order of the electron number density. It is estimated that [15]

$$R_n = \begin{cases} 0.024, & \text{mantle} \\ 0.146, & \text{core} \end{cases} \quad (17)$$

where $R_n = (n_n - n_e)/n_e$. So we can conclude that for energy < 1 GeV we have $|\Delta m_{31}^2/(2E)| \gg |V_n|$. From this conclusion we can find from Eqs. (9) and (16) that the correction to the probability of the ν_s to ν'_e or ν'_μ conversion through ν'_τ and vice versa, i.e.

$$\nu_s \rightleftharpoons \nu'_\tau \rightleftharpoons (\nu'_e, \nu'_\mu), \quad (18)$$

is suppressed by factor

$$\sin 2\theta_{03} \frac{2EV_e}{\Delta m_{31}^2} \sin 2\theta_{13} \quad \text{or} \quad \sin^2 2\theta_{03}. \quad (19)$$

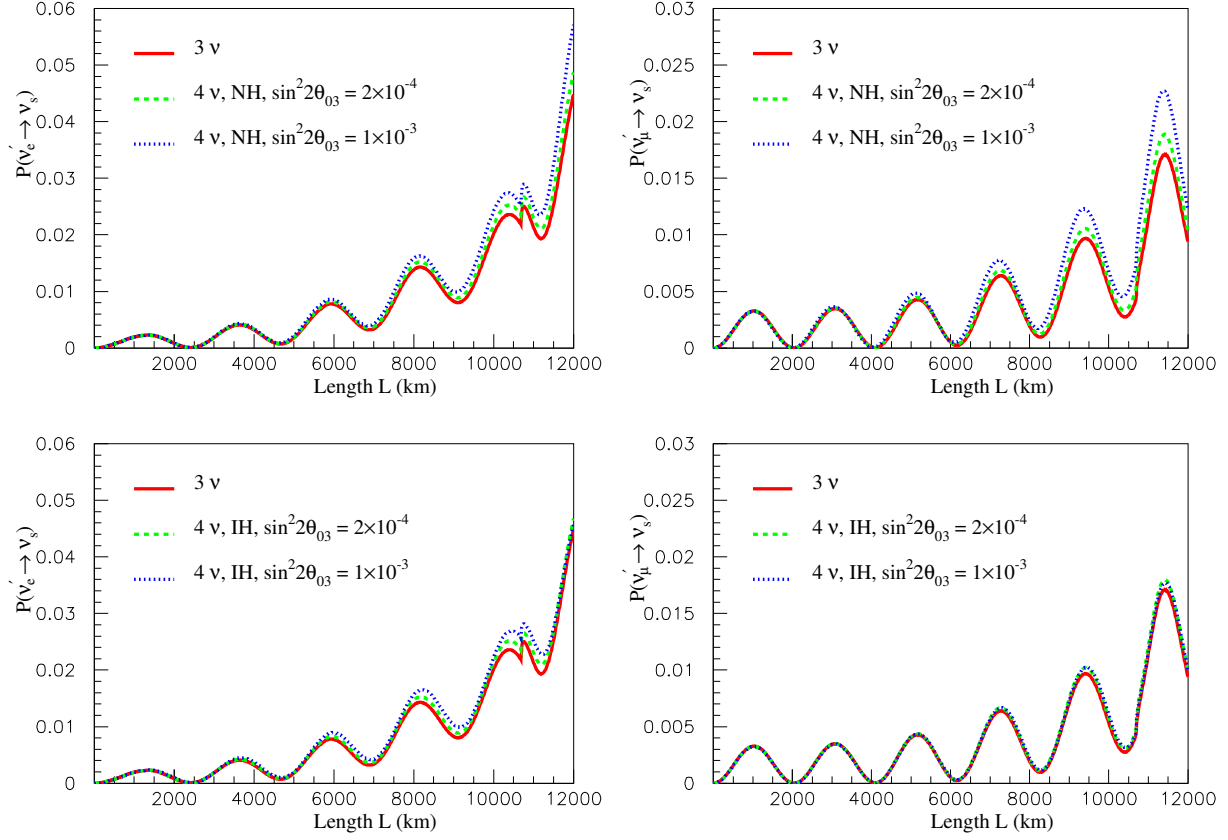


FIG. 1: $\nu'_e - \nu_s$ and $\nu'_\mu - \nu_s$ conversion probability in three neutrino framework v.s. in four neutrino framework. $E = 60$ MeV, $\Delta\hat{m}_{01}^2 = 0.5 \times 10^{-5}$ eV², $\sin^2 2\theta_{02} = 0.005$, $\sin^2 2\theta_{01} = 0$.

For $\sin^2 2\theta_{13} \approx 0.09$ [12–14] and $\sin^2 2\theta_{03} \lesssim 0.001$, this amplitude is maximally of order 10^{-3} for $E < 1$ GeV. Hence the effects of $\nu_s - (\nu'_e, \nu'_\mu)$ oscillation through ν'_τ is sub-leading and can be neglected.

In Fig. 1 we give the plots of $\nu_s - (\nu'_e, \nu'_\mu)$ conversion calculated in the 4ν framework using Hamiltonian (9) and in the 3ν framework using \hat{H}' in Hamiltonian (9) respectively. Relevant neutrino parameters in our calculation are [11]

$$\Delta m_{21}^2 = (7.50 \pm 0.20) \times 10^{-5} \text{ eV}^2, \quad |\Delta m_{32}^2| = (2.32^{+0.12}_{-0.08}) \times 10^{-3} \text{ eV}^2, \quad (20)$$

$$\sin^2 2\theta_{12} = 0.857 \pm 0.024, \quad \sin^2 2\theta_{23} > 0.95. \quad (21)$$

Soon after the discovery of a not small θ_{13} by Daya-Bay collaboration [12], confirmed by RENO experiment [13], a precise measurement of θ_{13} has been achieved by Daya-Bay

experiment [14]:

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005. \quad (22)$$

We use $\sin^2 2\theta_{23} = 1$ in our calculation. For other parameters we use the central values in Eqs. (20), (21) and (22). Results in Fig. 1 have been shown both for the case of normal hierarchy(NH) and for the case of inverted hierarchy(IH).

In Fig. 1 we can see that the result calculated in 3ν framework agrees with that calculated in 4ν framework in the case that $\sin^2 2\theta_{03}$ is small. Actually, for $\theta_{03} = 0$ two lines for 4ν and 3ν overlap. We see that a non-zero but small θ_{03} can not change the conversion qualitatively. This agrees with our discussion presented above.

In the following we will discuss effects of θ_{01} and θ_{02} in neutrino oscillation and will set $\theta_{03} = 0$. So we can study the oscillation of $\nu_s - (\nu'_e, \nu'_\mu)$ effectively in a 3ν framework

$$i\frac{d}{dt}\hat{\nu}' = \hat{H}\hat{\nu}', \quad (23)$$

where $\hat{\nu}' = (\nu_s, \nu'_e, \nu'_\mu)^T$. \hat{H} is given by

$$\hat{H} = \hat{U}\hat{H}_0\hat{U}^\dagger + \hat{V}, \quad (24)$$

with \hat{H}_0 obtained from (11) as

$$\hat{H}_0 = \text{diag}\left\{\frac{\Delta\hat{m}_{01}^2}{2E}, 0, \frac{\Delta m_{21}^2}{2E}\right\}, \quad (25)$$

where

$$\Delta\hat{m}_{01}^2 = \Delta m_{01}^2 \cos^2 \theta_{03} + \Delta m_{31}^2 \sin^2 \theta_{03}, \quad (26)$$

and

$$\hat{V} = \text{diag}\{0, V_e + V_n, V_n\}. \quad (27)$$

(27) is obtained from (12) by approximating $\cos^2 \theta_{13} = 1$. Since $\sin^2 2\theta_{13} \approx 0.09$ and $\sin^2 \theta_{13} \approx 0.022$ this is a valid approximation. $\Delta\hat{m}_{01}^2$ is basically the parameter governing the solar ν_e disappearance to ν_s discussed in [2]. In the following we will set $\theta_{03} = 0$. Hence we will not differentiate between Δm_{01}^2 and $\Delta\hat{m}_{01}^2$ in the following of the present article.

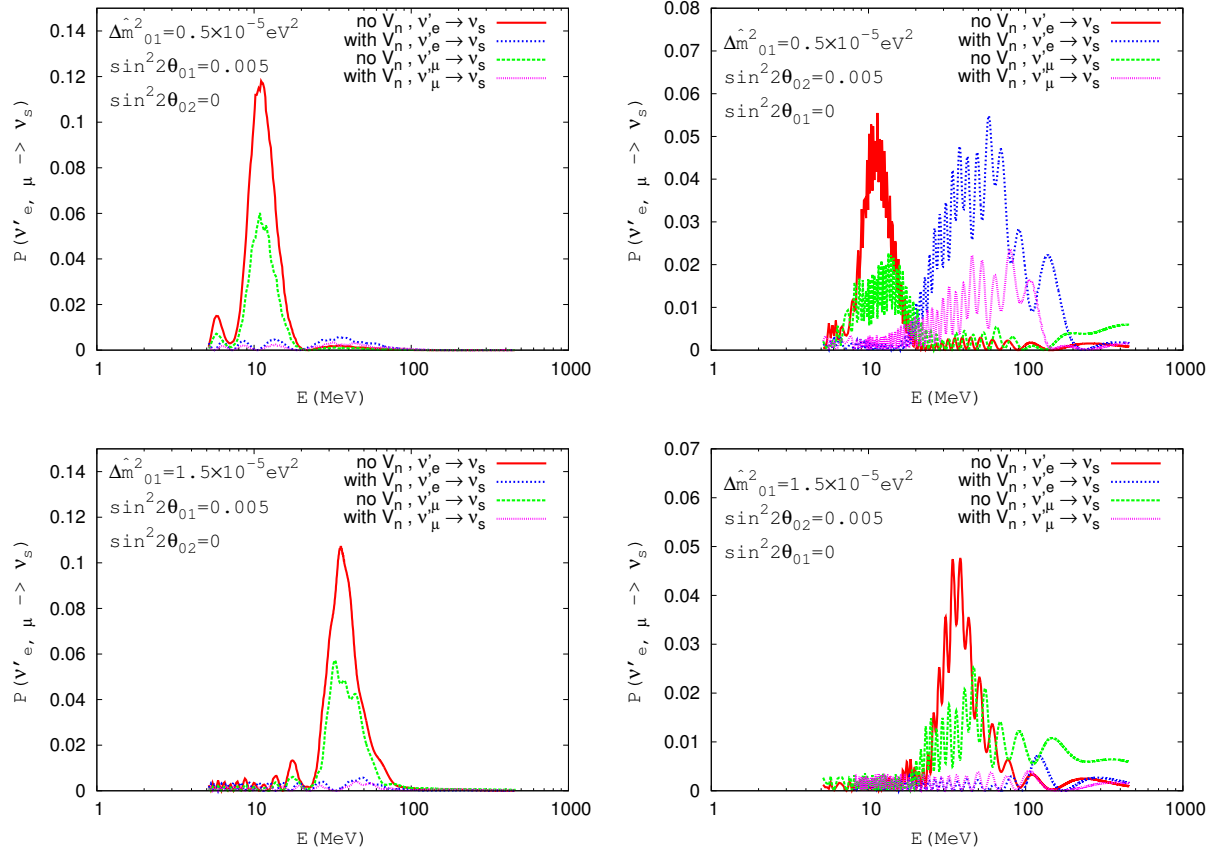


FIG. 2: $\nu'_e - \nu_s$ and $\nu'_\mu - \nu_s$ conversion probability versus energy in three neutrino framework both for V_n included and for V_n switched off. $L = 12000$ km.

Fig. 1 is given with the effect of V_n included and has been shown to a particular energy that $\nu_s - \nu'_e$ conversion is close to maximal. In Fig. 2 we give plots of the conversion probability of $\nu'_{e,\mu} \rightarrow \nu_s$ versus energy both for V_n included and for V_n switched off. For the case with the effect of V_n included the neutron number density is calculated using (17). We can see that for the case with the effect of V_n included the probability of $\nu'_e \rightarrow \nu_s$ conversion is maximally around 5% for $\sin^2 2\theta_{02} = 0.005$. For smaller $\sin^2 2\theta_{02}$ the amplitude of the conversion probability is smaller. This enhancement of conversion probability happens for $\Delta\hat{m}_{01}^2$ around $0.5 \times 10^{-5} \text{ eV}^2$ and for energy around 60 MeV. For $\Delta\hat{m}_{01}^2 = 1.5 \times 10^{-5} \text{ eV}^2$, the conversion probability is maximally around 10^{-3} and the resonant conversion disappears.

In contrast, we can see in Fig. 2 that there are indeed much stronger resonant $\nu_s - (\nu'_e, \nu'_\mu)$ conversions when V_n is switched off. In particular, there is a resonance conversion

for energy around 10 MeV when $\Delta\hat{m}_{01}^2$ is around $0.5 \times 10^{-5} \text{ eV}^2$. This is consistent with our expectation. Fortunately, this resonance disappears after including effect of V_n . Although there is still a resonant enhancement of $\nu'_{e,\mu} \rightarrow \nu_s$ conversion for $\Delta\hat{m}_{01}^2 = 0.5 \times 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta_{02} = 0.005$ when effect of V_n included, the conversion probability can only reach about 5% and furthermore it appears at energy around 60 MeV which is well beyond the solar neutrino and supernovae neutrino spectrum. For larger $\Delta\hat{m}_{01}^2$ this enhancement disappears completely, as can be seen in the plot for $\Delta\hat{m}_{01}^2 = 1.5 \times 10^{-5} \text{ eV}^2$ in Fig. 2. More details about the variation with respect to $\Delta\hat{m}_{01}^2$ will be presented in the next section.

This is a very interesting observation. If the resonant conversion happens as in the case with V_n switched off, active neutrinos of energy around 10 to 60 MeV can oscillate to this super-light sterile. In particular, this could lead to spectrum distortion of high energy solar boron neutrino and the flux of low energy atmospheric active neutrinos should have a dip at this energy range. The disappearance or the suppression of the resonant conversion in the case with V_n included shows that the scenario of super-light sterile neutrino can pass through the possible constraints from solar and atmospheric neutrino experiments.

We note that this phenomenon does not happen for the conversion of neutrino in the Sun since the neutron number density is small in the Sun and the effect of V_n is indeed switched off. On the other hand, the neutron number density is of the same order of the magnitude of the electron number density in the Earth and the effect of V_n can play important role. In the next section we show in detail how the level crossing and the resonance disappear in the Earth matter.

Level crossing and flavor conversion of the super-light sterile neutrino

In this section we study the effect of V_n on the level crossing and the resonant conversion of the super-light sterile neutrino ν_s with active neutrinos ν'_e and ν'_μ in the Earth. We will work in the 3ν framework as given in Eqs. (23), (24), (25), (26) and (27).

A MSW resonance of flavor conversion happens when two energy levels become close to each other. If a resonance happens a small mixing in vacuum can effectively lead to large flavor conversion in matter. This happens in particular for the case that there is a crossing of two energy levels of the Hamiltonian. To examine whether there is a level crossing or a resonance it's crucial to check whether two eigenvalues of the Hamiltonian \hat{H}

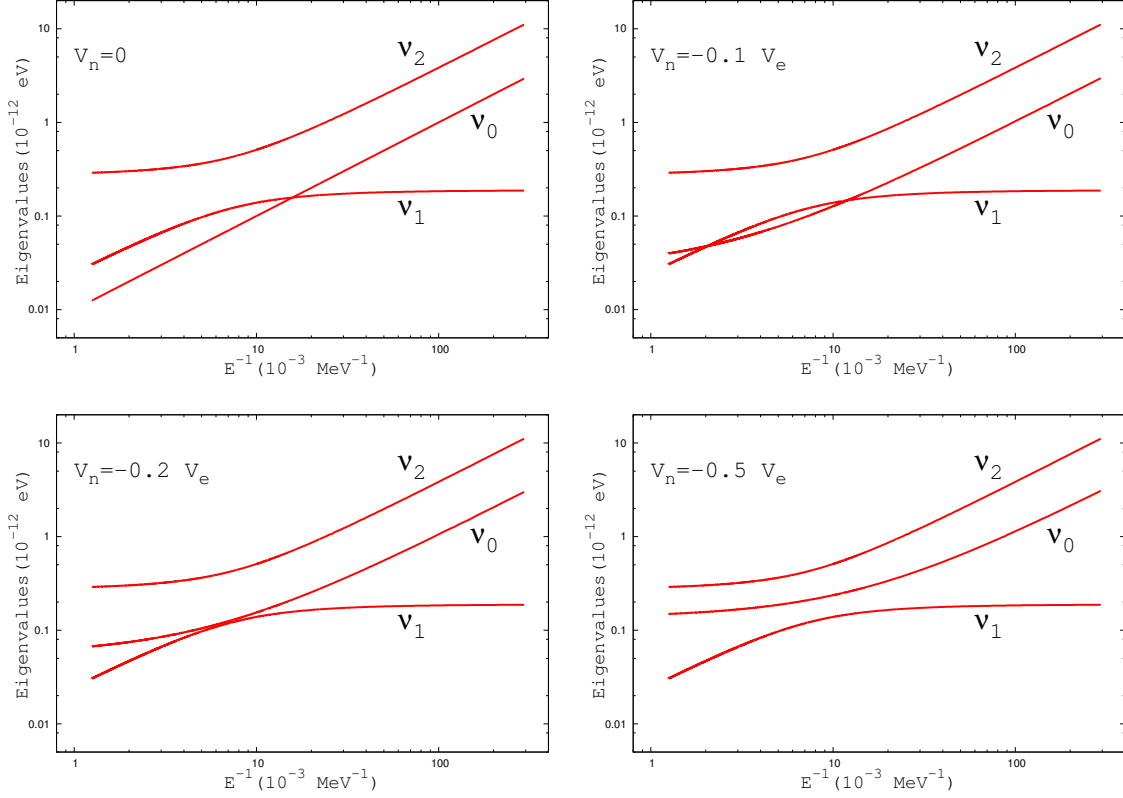


FIG. 3: Energy levels ν_0 , ν_1 and ν_2 in three neutrino framework. Three lines for ν_0 , ν_1 and ν_2 are $E_0 - V_n$, $E_1 - V_n$ and $E_2 - V_n$ separately. $\theta_{01} = \theta_{02} = 0$, $\Delta\hat{m}_{01}^2 = 2 \times 10^{-5} \text{ eV}^2$, $\Delta m_{21}^2 = 7.5 \times 10^{-5} \text{ eV}^2$, $\sin^2 2\theta_{12} = 0.857$, $V_e = 2.74 \times 10^{-13} \text{ eV}$.

become close to each other. As will be shown below, it's sufficient to set small parameters θ_{01} and θ_{02} to zero when studying the energy levels of \hat{H} . For convenience we will set $\theta_{01} = \theta_{02} = 0$ and discuss the energy levels of the following Hamiltonian

$$\hat{H} = \begin{pmatrix} \frac{\Delta\hat{m}_{01}^2}{2E} & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} \sin^2 \theta_{12} + V_e + V_n & \frac{\Delta m_{21}^2}{2E} \sin \theta_{12} \cos \theta_{12} \\ 0 & \frac{\Delta m_{21}^2}{2E} \sin \theta_{12} \cos \theta_{12} & \frac{\Delta m_{21}^2}{2E} \cos^2 \theta_{12} + V_n \end{pmatrix}. \quad (28)$$

(28) is obtained from (24) by setting $\theta_{01} = \theta_{02} = 0$ in \hat{U} .

The convenience of using (28) is clear by noting that the three eigenvalues of (28) can

be easily found as

$$E_0 = \frac{\Delta\hat{m}_{01}^2}{2E}, \quad (29)$$

$$E_1 = \frac{1}{2} \left[\frac{\Delta m_{21}^2}{2E} + V_e - \sqrt{\left(\frac{\Delta m_{21}^2}{2E} \cos 2\theta_{12} - V_e \right)^2 + \left(\frac{\Delta m_{21}^2}{2E} \sin 2\theta_{12} \right)^2} \right] + V_n, \quad (30)$$

$$E_1 = \frac{1}{2} \left[\frac{\Delta m_{21}^2}{2E} + V_e + \sqrt{\left(\frac{\Delta m_{21}^2}{2E} \cos 2\theta_{12} - V_e \right)^2 + \left(\frac{\Delta m_{21}^2}{2E} \sin 2\theta_{12} \right)^2} \right] + V_n. \quad (31)$$

E_0 , E_1 , E_2 are eigenvalues of (28) corresponding to neutrinos in mass base ν_0 , ν_1 and ν_2 separately. To illustrate qualitatively the effect of the Earth matter on the oscillation of super-light sterile neutrino ν_s with active neutrinos ν'_e and ν'_μ , it's convenient to compute E_1 and E_2 using a trajectory dependent averaged potential [16]

$$\bar{V}_e = \frac{1}{L} \int_0^L dx V_e(x), \quad (32)$$

where L is the length of the neutrino trajectory in the Earth. For baseline longer than 6000 km, \bar{V}_e varies from 1.6×10^{-13} eV to about 2.7×10^{-13} eV. As will be detailed in the next section a formulation using the average potential (32) gives a pretty good description of the oscillation of super-light sterile neutrino with active neutrinos.

In Fig. 3 we give plots for $E_i - V_n$ and compare different cases with various V_n . We can see that for $V_n = 0$ there is indeed a level crossing of E_0 and E_1 at energy around 50 – 60 MeV when $\Delta\hat{m}_{01}^2 = 2 \times 10^{-5}$ eV² and there should be a resonant conversion associated with it, as has been shown in Fig. 2 for $\Delta\hat{m}_{01}^2 = 1.5 \times 10^{-5}$ eV². Increasing the magnitude of V_n first gives rise to a second point of level crossing, as can be seen in the plot with $V_n = -0.1V_e$. But for $V_n = -0.5V_e$ there is no crossing point of two lines of E_0 and E_1 and the resonance disappears. We note that this is exactly the situation we have in the Earth matter. As can be seen in Eq. (17), the neutron number density roughly equals to the electron number density in the Earth and hence $V_n \approx -0.5V_e$.

For a non-zero but small θ_{01} or θ_{02} one can give similar plots for E_0 , E_1 and E_2 and there are no visible differences compared to the plots in Fig. 3. So the above discussion presented for $\theta_{01} = \theta_{02} = 0$ can be applied to the case with non-zero but small θ_{01} or θ_{02} and the reason of the absence of resonance for a relatively large value of $\Delta\hat{m}_{01}^2$ is clear according to discussion presented above. In Fig. 4 we give plots of $E_i - V_n$ for $\theta_{01,02} \neq 0$. E_i are obtained from diagonalizing (24) numerically. V_n/V_e is fixed in Fig. 4 while $\Delta\hat{m}_{01}^2$

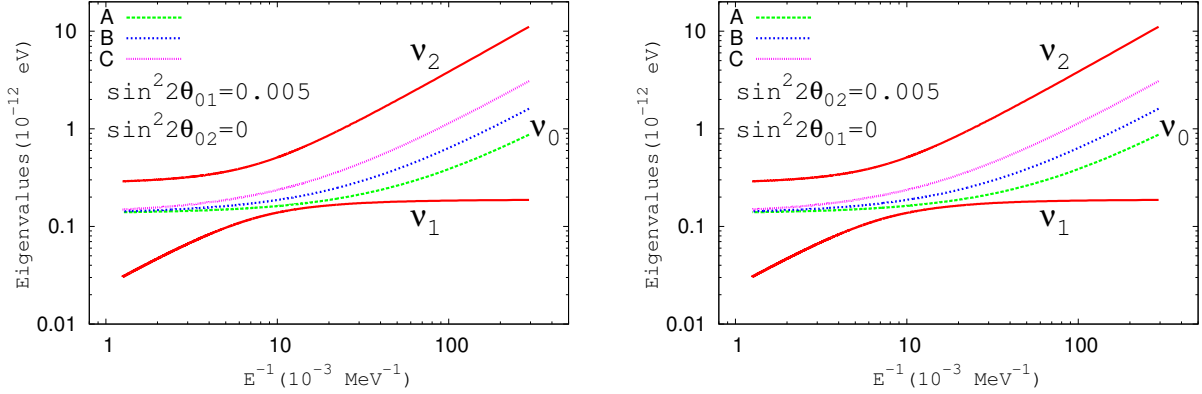


FIG. 4: $E_0 - V_n$, $E_1 - V_n$ and $E_2 - V_n$ for ν_0 , ν_1 and ν_2 in three neutrino framework. $V_n = -0.5V_e$ and $V_e = 2.74 \times 10^{-13}$ eV. Lines A, B and C correspond to cases with $\Delta\hat{m}_{01}^2 = 0.5 \times 10^{-5}$ eV², $\Delta\hat{m}_{01}^2 = 1.0 \times 10^{-5}$ eV² and $\Delta\hat{m}_{01}^2 = 2.0 \times 10^{-5}$ eV² separately.

varies. In Earth matter V_e/V_n can vary from -0.5012 to -0.573 . As can be seen in Fig. 3, the larger the $|V_e/V_n|$ is, the farther away from the level crossing the two energy levels. So we choose $V_e = -0.5V_n$ in Fig. 4. We can see that for $\Delta\hat{m}_{01}^2 = (0.5 - 2) \times 10^{-5}$ eV² the line of E_0 is always in-between the two lines of E_1 and E_2 . The only possible case for a resonant conversion to happen is when $\Delta\hat{m}_{01}^2 \approx 0.5 \times 10^{-5}$ eV². In this case E_0 and E_1 come close to each other although there is no crossing. However, even in this case the $\nu'_{e,\mu} \rightarrow \nu_s$ conversion probability is maximally around 5% when $\sin^2 2\theta_{02} = 0.005$, as has been shown in Fig. 2.

In Fig. 5 we give plots of $E_i - V_n$ similar to Fig. 4 but with different V_e . We can see that the situation is very similar to that in Fig. 4 and there are no level crossing of E_0 and E_1 energy levels. From the above discussion we can conclude that due to the presence of V_n in Earth matter the level crossing expected for E_0 and E_1 disappears.

In Fig. 6 we show the variation of the probability of $\nu'_{e,\mu} \rightarrow \nu_s$ conversion with respect to $\Delta\hat{m}_{01}^2$. We can see that as $\Delta\hat{m}_{01}^2$ increases from the value 0.5×10^{-5} eV², the amplitude of the conversion probability decreases and for $\Delta\hat{m}_{01}^2 > 1.0 \times 10^{-5}$ eV² the probability is maximally around 1%. We see that the resonant conversion disappears for a relatively large value of $\Delta\hat{m}_{01}^2$.

For solar neutrinos one can similarly show that if $n_n/n_e \gtrsim 0.9$ in the Sun the level

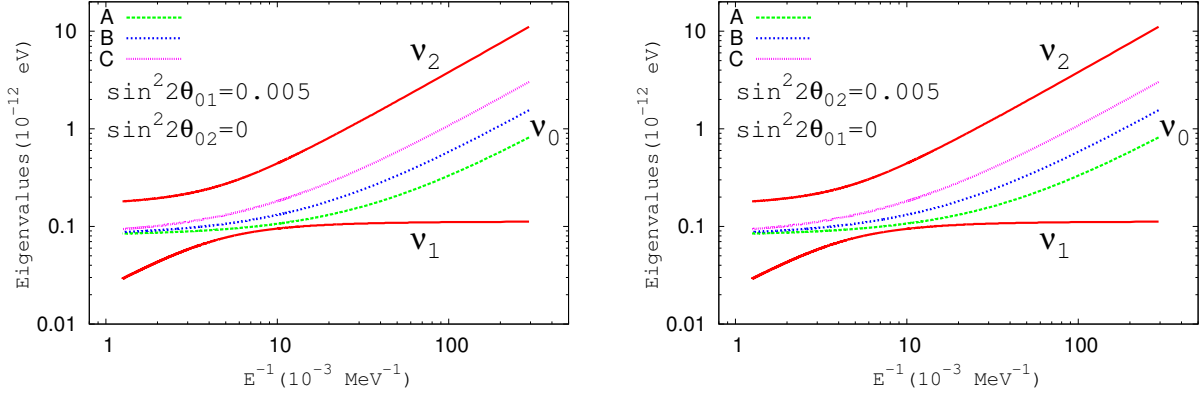


FIG. 5: $E_0 - V_n$, $E_1 - V_n$ and $E_2 - V_n$ for ν_0 , ν_1 and ν_2 in three neutrino framework. $V_n = -0.5V_e$ and $V_e = 1.63 \times 10^{-13}$ eV. Lines A, B and C correspond to cases with $\Delta\hat{m}_{01}^2 = 0.5 \times 10^{-5}$ eV², $\Delta\hat{m}_{01}^2 = 1.0 \times 10^{-5}$ eV² and $\Delta\hat{m}_{01}^2 = 2.0 \times 10^{-5}$ eV² separately.

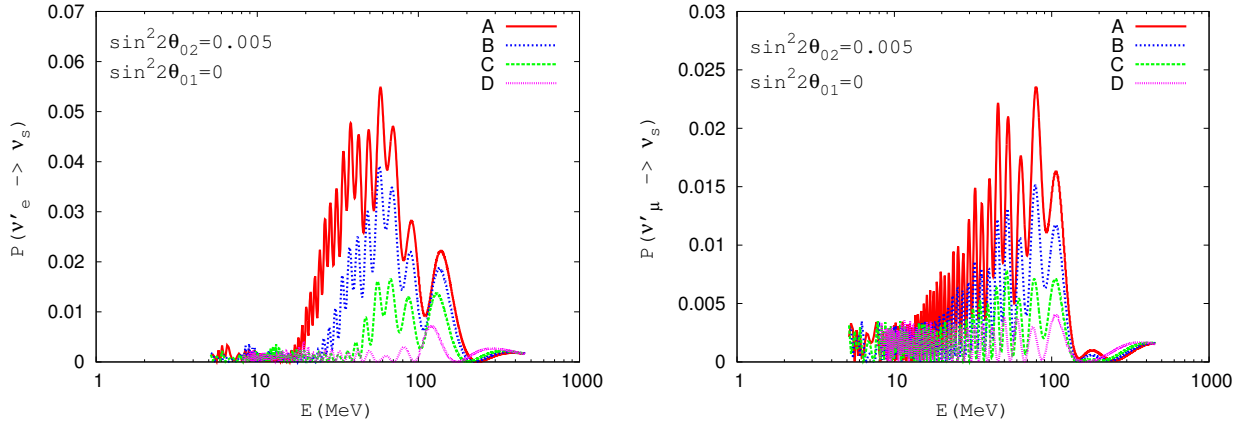


FIG. 6: $\nu'_e \rightarrow \nu_s$ and $\nu'_\mu \rightarrow \nu_s$ conversion probability versus energy. Lines A, B, C and D correspond to $\Delta\hat{m}^2 = 0.5 \times 10^{-5}$ eV², $\Delta\hat{m}^2 = 0.7 \times 10^{-5}$ eV², $\Delta\hat{m}^2 = 1.0 \times 10^{-5}$ eV², $\Delta\hat{m}^2 = 1.5 \times 10^{-5}$ eV² separately. $L = 12000$ km.

crossing would disappear. For real matter density in the Sun which has a negligible neutron number density, the level crossing and the MSW resonance indeed exist.

Formulation of oscillation of super-light sterile neutrino in the Earth

In this section we present a perturbation theory describing the oscillation of super-light sterile neutrino with active neutrinos in the Earth matter. This formulation uses the baseline averaged potential (32) and its basic strategy is the same as that presented

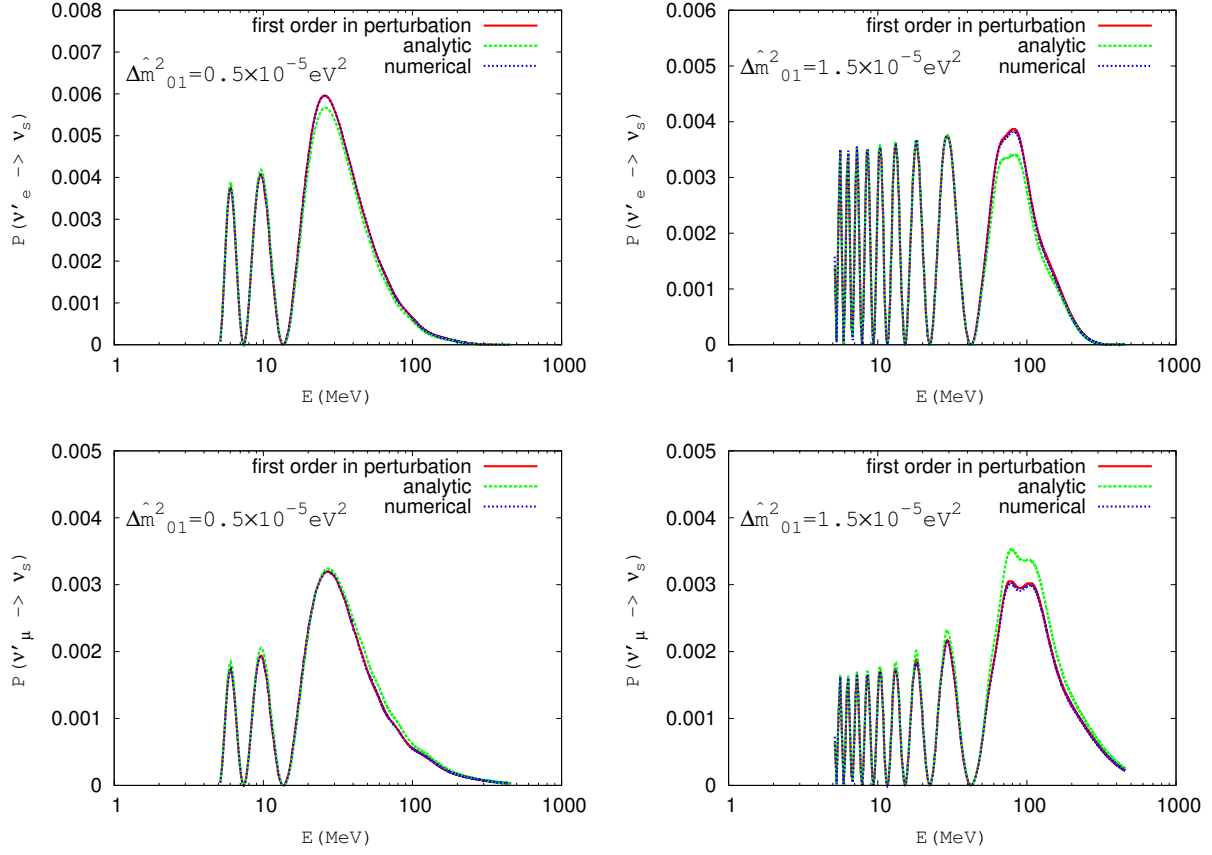


FIG. 7: $P(\nu'_{e,\mu} \rightarrow \nu_s)$ versus energy for $L = 8000$ km. $\sin^2 2\theta_{01} = 0.005$, $\theta_{02} = 0$.

for oscillation among active neutrinos in [16]. We will show that the leading term in the theory, which is analytic, gives a qualitatively good description of the flavor conversion of the super-light sterile neutrino ν_s with the active neutrinos. Including the first order correction this perturbation theory is precise to describe the flavor conversion between ν_s and $\nu'_{e,\mu}$. This justifies the use of the average potential (32) in the discussion of the level crossing of energy levels in the last section. The theory is detailed below.

Similar to the formulation in [16], we can introduce an average potential for a trajectory of neutrino with a baseline length L in the Earth

$$\bar{V} = \text{diag}\{0, \bar{V}_e + \bar{V}_n, \bar{V}_n\} = \frac{1}{L} \int_0^L dx \hat{V}(x), \quad (33)$$

where \bar{V}_e has been introduced in (32) and \bar{V}_n is similarly defined. The Hamiltonian (24) can be rewritten using (33) as

$$\hat{H}(x) = \bar{H} + \delta\hat{H}(x), \quad (34)$$

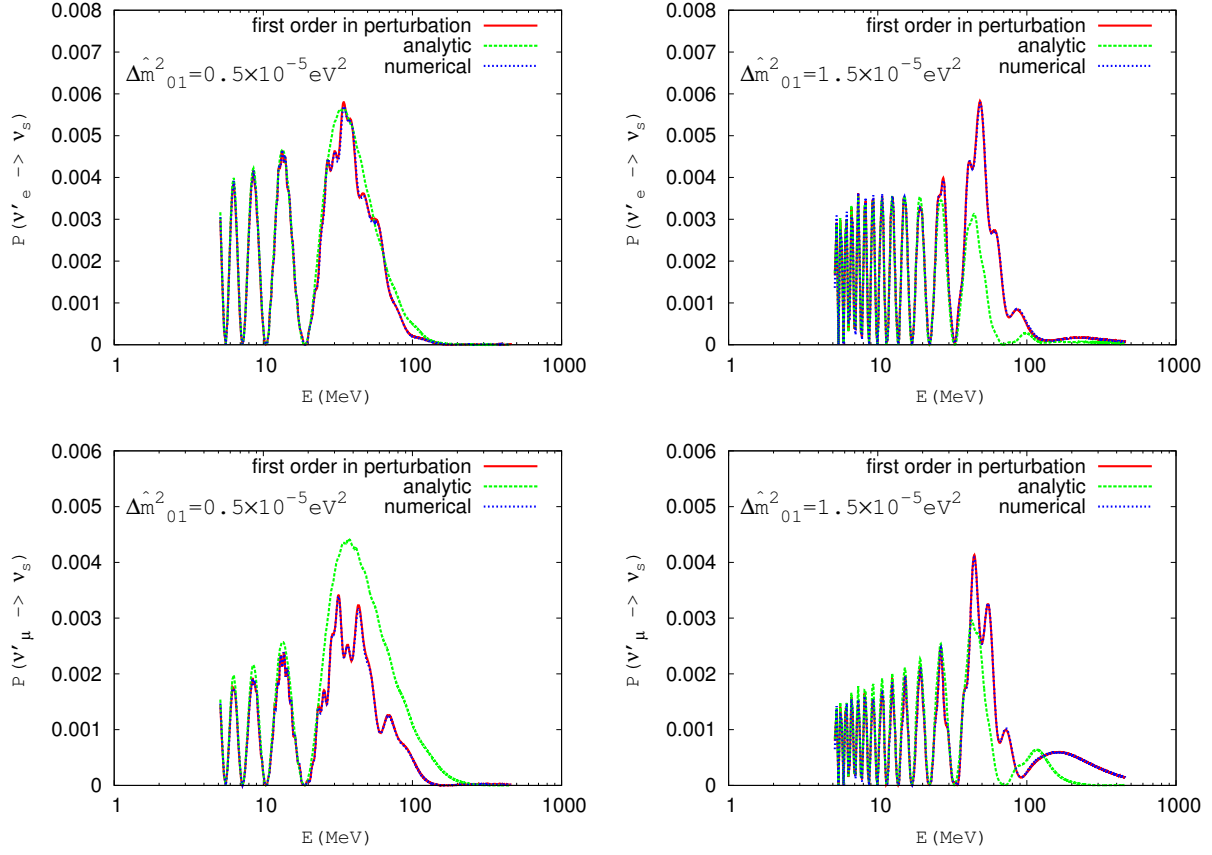


FIG. 8: $P(\nu'_e, \nu'_\mu \rightarrow \nu_s)$ versus energy for $L = 12000$ km. $\sin^2 2\theta_{01} = 0.005$, $\theta_{02} = 0$.

where

$$\bar{H} = \hat{U} \hat{H}_0 \hat{U}^\dagger + \bar{V}, \quad (35)$$

and $\delta \hat{H} = \hat{H} - \bar{H}$ which equals to $\delta \hat{V}$

$$\delta \hat{H} = \delta \hat{V} = \hat{V} - \bar{V}. \quad (36)$$

Introducing a mixing matrix U_m in matter which diagonalizes \bar{H} :

$$\bar{H} U_m = U_m \frac{1}{2E} \Delta, \quad \Delta = \text{diag}\{\Delta^1, \Delta^2, \Delta^3\}, \quad (37)$$

where $\frac{1}{2E} \Delta^i (i = 1, 2, 3)$ are three eigenvalues of \bar{H} , we are ready to solve the evolution problem in (23) perturbatively in an expansion in $\delta \hat{V}$. We first solve the evolution governed by \bar{H} and obtain the contribution of δV using perturbation in δV . Keeping result

of first order in δV we obtain

$$\widehat{\nu}'(L) = M(L)\widehat{\nu}'(0), \quad (38)$$

$$M(L) = U_m e^{-i\frac{\Delta}{2E}L}(1 - iC) U_m^\dagger, \quad (39)$$

where C is a 3×3 matrix accounting for the non-adiabatic correction

$$C = \int_0^L dx e^{i\frac{\Delta}{2E}x} U_m^\dagger \delta V(x) U_m e^{-i\frac{\Delta(x)}{2E}x}. \quad (40)$$

It is clear that $C^\dagger = C$ holds. One can see that

$$C_{jj} = \int_0^L dx (U_m^\dagger \delta V(x) U_m)_{jj} = 0, \quad (41)$$

$$C_{jk} = \int_0^L dx e^{i\frac{\Delta^j - \Delta^k}{2E}x} (U_m^\dagger \delta V(x) U_m)_{jk}, \quad j \neq k. \quad (42)$$

Eq. (41) is guaranteed by Eq. (33). $|C_{jk}| \ll 1 (j \neq k)$ should be satisfied if this is a good perturbation theory. One of the virtues of this perturbation theory is that Eq. (41) guarantees that the oscillation phase is correctly reproduced.

In Fig. 7 we give plots for $\nu_s - \nu'_e$ conversion versus energy for $L = 8000$ km. We give zero-th order results calculated using analytic formula, i.e. using (38) with C switched off. We also present the result including the first order correction (40). The line for numerical result is calculated using the PREM Earth density profile [17]. We can see that the zero-th order result using trajectory averaged potential reproduces the oscillation phase very well, but not the magnitude. Including the first order correction calculated using (40), this formalism reproduces very well the oscillation phase and magnitude of the flavor conversion.

When $L > 10690$ km neutrinos cross the core of the Earth. Large density jump between the core and the mantle makes the above simple version of the perturbation theory not as precise as shown in Fig. 7. We can improve the approximation by dividing the whole trajectory into three parts with parts 1 and 3 in the mantle and part 2 in the core of the Earth. The evolution matrix can be written as

$$M = M_3 M_2 M_1, \quad (43)$$

where M_2 is the evolution matrix in the core and $M_{1,3}$ are evolution matrices in the mantle. $M_i (i = 1, 2, 3)$ are calculated similar to (38) but with average potentials evaluated in the

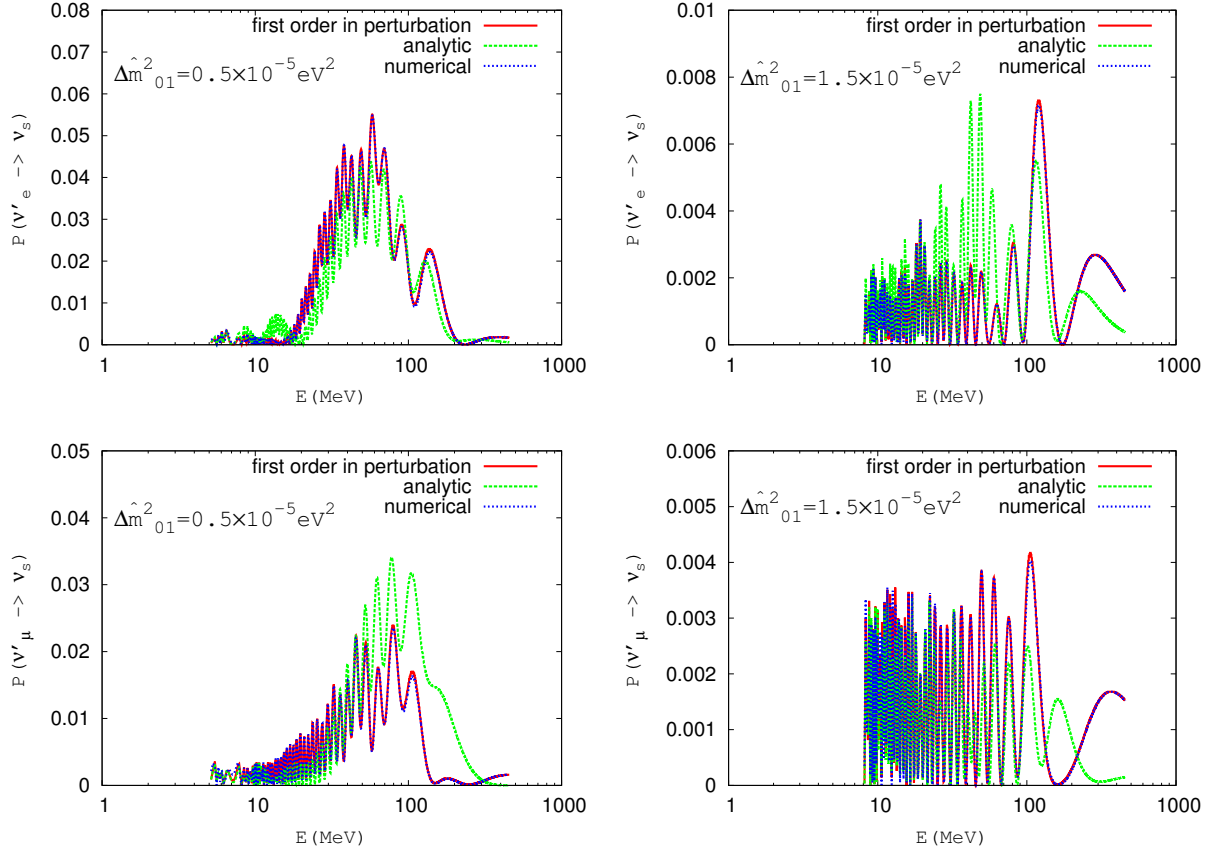


FIG. 9: $P(\nu'_{e,\mu} \rightarrow \nu_s)$ versus energy for $L = 12000$ km. $\sin^2 2\theta_{02} = 0.005$, $\theta_{01} = 0$.

corresponding part of the trajectory in the mantle and in the core separately [16]. In Fig. 8 we give plots for an example of this case. We can see that the result calculated using the improved perturbation theory (43) agrees well with the numerical result. The oscillation phase and the magnitude of the flavor conversion are very well reproduced in the perturbation theory. We also give the analytic result calculated using the potential averaged over the whole trajectory (33). We can see that this analytic result correctly reproduces the oscillation phase of the $\nu_s - \nu'_e$ conversion.

In Fig. 9 and 10 we give plots of $\nu'_{e,\mu} \rightarrow \nu_s$ oscillation similar to Fig. 7 and 8, but for non-zero θ_{02} . Similarly, we can see that the analytic result gives a good account of the phase of the oscillation of the super-light sterile neutrino with active neutrinos. The perturbation theory gives not only reproduces the oscillation phase but also the magnitude of the flavor conversion very well.

We note that the above discussion does not mean that the perturbation theory pre-

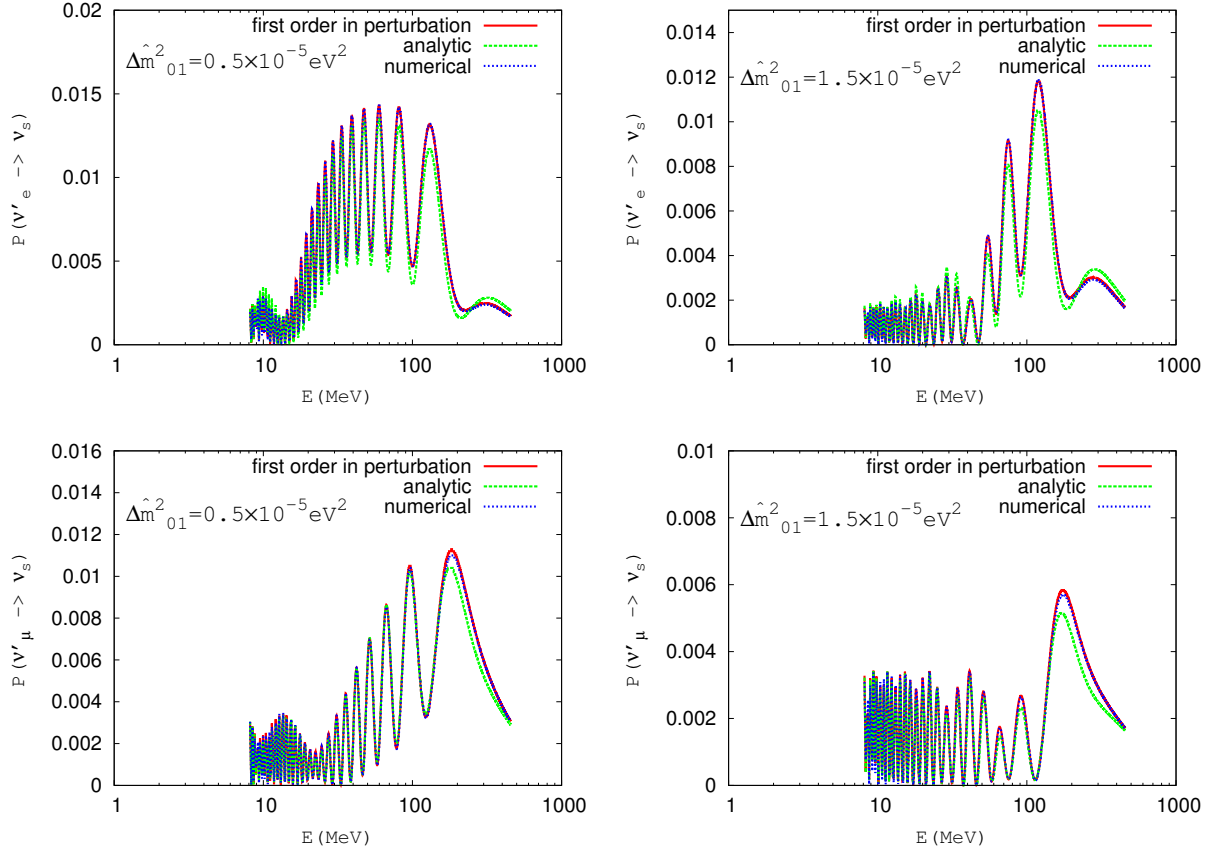


FIG. 10: $P(\nu'_e, \nu'_\mu \rightarrow \nu_s)$ versus energy for $L = 8000$ km. $\sin^2 2\theta_{02} = 0.005$, $\theta_{01} = 0$.

sented in this article give a precise description for $1 - 2$ oscillation. The theory presented in this article indeed give a qualitatively good description for $1 - 2$ oscillation for energy $\gtrsim 20$ MeV, but not precisely. Actually for an energy around 10 MeV the structure of the Earth matter shows up in $1 - 2$ oscillation [18] and the perturbation theory using average potential is not a precise description. This perturbation theory gives a precise description of $1 - 2$ oscillation for energy $\gtrsim 500$ MeV [16].

We can see from the above discussion that the zero-th order result calculated using the trajectory averaged potential, Eq. (33), always give a correct account of the oscillation phase of the $\nu_s - \nu'_{e,\mu}$ conversion. Although the zero-th order result does not give a precise description of the magnitude of conversion, it encodes the major properties of the flavor conversion in the Earth. This justifies the use of the average potential in the discussion of the level crossing and the disappearance of the resonance in the last section.

Conclusion:

In summary we have made a detailed study of the flavor conversion of the super-light sterile neutrino with active neutrinos in Earth matter. A super-light sterile neutrino, with a mass squared difference $\Delta m_{01}^2 \approx (0.5 - 2) \times 10^{-5} \text{ eV}^2$ and a small mixing angle θ_{01} or θ_{02} , can oscillate with electron neutrinos in the Sun through a MSW resonance and can help to explain the absence of the upturn of the solar boron neutrino spectrum at energy $\lesssim 4 \text{ MeV}$. One would naively expect that a similar resonant conversion should also happen when neutrinos pass through the Earth. In this article we have shown that for $\Delta m_{01}^2 \gtrsim 1 \times 10^{-5} \text{ eV}^2$ this naively expected resonant conversion disappears completely and for smaller value of Δm_{01}^2 there is still an enhancement of the flavor conversion but the conversion probability is at most a few percent.

We have shown that the absence or the suppression of the resonant conversion is because of the presence of the potential V_n which arises from neutral current interaction of active neutrinos with neutrons in matter. The neutron number density in the Sun is negligible comparing to the electron density and the effect of V_n is basically switched off. On the other hand, the neutron number density in the Earth roughly equals to the electron number density and the effect of V_n can play important role. We have shown that the naively expected level crossing of energy levels of neutrinos disappears when including the effect of V_n in Earth matter. In particular, we find that for $\Delta m_{01}^2 = (0.5 - 2) \times 10^{-5} \text{ eV}^2$ the energy of the super-light neutrino is always in-between the energies of ν_1 and ν_2 neutrinos and there is no crossing of energy levels among these neutrinos. For Δm_{01}^2 around $0.5 \times 10^{-5} \text{ eV}^2$ the energies of the super-light sterile neutrino and the active neutrino ν_1 have a chance to be close to each other when the energy is around 60 MeV and hence create a resonant conversion. However this resonant enhancement of the flavor conversion is significantly suppressed comparing to the case when V_n is switched off and the flavor conversion probability is at most a few percent. Furthermore, the position of the resonance is shifted to an energy around 60 MeV which makes this scenario easily be able to escape possible constraints coming from measurement of Earth matter effect in solar neutrino experiments.

Apparently the scenario of super-light sterile neutrino is difficult to test in ground-based experiment of neutrino oscillation. We have shown that the conversion of $\nu'_{e,\mu} \rightarrow \nu_s$ is maximally a few percent and the maximal conversion happens for energy around 60 MeV.

This energy range is well beyond that of the solar and supernovae neutrino spectrum. The only possibility is to test it in very low energy atmospheric neutrino data. However, this is also pretty difficult because the conversion probability is maximally a few percent but there are quite a lot of uncertainties in low energy atmospheric neutrinos.

For completeness of our discussion we have also shown a perturbation theory which makes use of an baseline averaged potential in developing the theory. This theory can very well describe the flavor conversion of the super-light sterile neutrino with active neutrinos in Earth matter. This justifies the use of the baseline averaged potential in the discussion of the level crossing of neutrinos. We have also discussed the reduction of the flavor conversion in the 4ν framework to a 3ν framework.

The outcome of the present research is interesting. A MSW resonance is needed in explaining the absence of the upturn of the solar boron neutrino at energy $\lesssim 4$ MeV [2]. However, a resonant conversion of this super-light sterile neutrino with active neutrino is dangerous when confronting this scenario with the experiments which can test the Earth matter effect, e.g. the solar and atmospheric neutrino experiments. Interestingly, we find that the effect of V_n in the Earth makes the resonant conversion disappeared or significantly suppressed and the scenario of the super-light sterile neutrino can pass through test of Earth matter effect in neutrino oscillation experiments. On the other hand, the effect of V_n in the Sun is actually switched off because of the negligible neutron number density and this enables the MSW resonance in the Sun. This is exactly the situation welcomed when confronting the scenario of the super-light sterile neutrino with oscillation experiments. It's interesting to see that it is the different situation in the Sun and in the Earth that makes V_n effectively turned off and turned on respectively. This observation seems to suggest that the super-light sterile neutrino is a natural choice made by the Sun and the Earth rather than by the authors of Refs. [1, 2]. It makes the scenario of the super-light sterile neutrino very interesting.

Acknowledgments

This work is supported by National Science Foundation of China(NSFC), grant No.11135009, No. 11375065 and Shanghai Key Laboratory of Particle Physics and Cos-

mology, grant No. 11DZ2230700.

-
- [1] P. C. de Holanda and A. Yu. Smirnov, Phys. Rev. **D69**,113002(2004).
 - [2] P. C. de Holanda and A. Yu. Smirnov, Phys. Rev. **D83**, 113011(2011).
 - [3] For lower bound on Δm_{01}^2 we take $0.5 \times 10^{-5} \text{ eV}^2$ which comes from a discussion of Fig. 7 in Ref. [2]. Taking the more conservative lower bound $0.7 \times 10^{-5} \text{ eV}^2$ does not change the conclusion in this article.
 - [4] B. T. Cleveland, et al., Astrophys.J. 496, 505 (1998).
 - [5] K. Abe et al., Phys. Rev. **D83**,052010(2011).
 - [6] B. Aharmim et al. [SNO Collaboration], Phys. Rev. **C81**,055504(2010).
 - [7] G. Bellini et al. [The Borexino Collaboration], Phys. Rev. **D82**,03300(2010).
 - [8] L. Wolfenstein, Phys. Rev. **D 17**, 2369 (1978); L. Wolfenstein, in "Neutrino-78", Purdue Univ. C3 - C6, (1978).
 - [9] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cim. **C9**, 17 (1986); S. P. Mikheyev and A. Yu. Smirnov, ZHETF, **91**, (1986), [Sov. Phys. JETP, **64**, 4 (1986)] (reprinted in "Solar neutrinos: the first thirty years", Eds. J.N.Bahcall *et. al.*).
 - [10] For other works discussing the Earth effect in oscillation of light sterile neutrino with active neutrinos, see e.g. H. W. Long, Y. F. Li, C. Giunti, JHEP **1308**, 056 (2013);H. W. Long, Y. F. Li, C. Giunti, Phys. Rev. **D87**, 113004(2013).
 - [11] Review of Particle Physics, Phys. Rev. **D86**, 010001(2012) and update online at pdg.lbl.gov.
 - [12] F. P. An et al. [Daya Bay collaboration], Phys. Rev. Lett.**108**, 171803(2012).
 - [13] J. K. Ahn et al.[RENO collaboration], Phys. Rev. Lett.**108**, 191802(2012).
 - [14] Daya Bay Collaboration (F. P. An et al.), Chinese Phys. C37, 011001(2013).
 - [15] E. Lisi, D. Montanino, Phys. Rev. **D56**, 1792(1997).
 - [16] W. Liao, Phys. Rev. **D77**, 053002(2008).
 - [17] A.M. Dziewonski and D.L. Anderson, 1875 Phys. Earth. Planet. Inter.**25**(1981)297.
 - [18] P. C. de Holanda, W. Liao, A. Yu. Smirnov, Nucl. Phys. B **702**, 307(2004).